Memory Effect of Gravity & Infrared Triangle

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ADVANCED TOPICS OF GRAVITY
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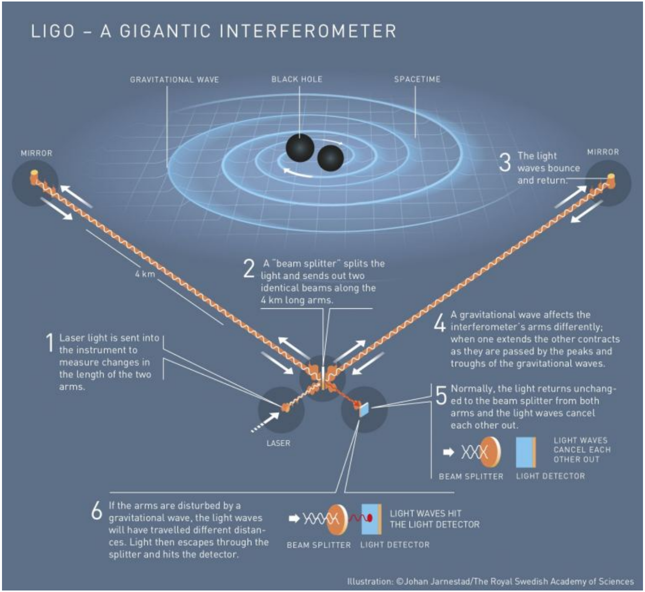
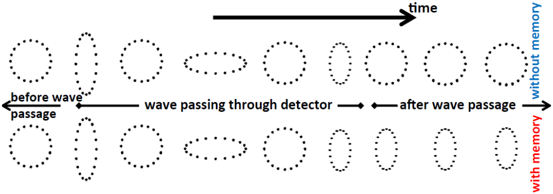
Gravitational Wave



Gravitational Wave Memory Effect

A subtle DC effect: passage of gravitational waves (GWs) produces a permanent shift in the relative positions of a pair of inertial detectors.

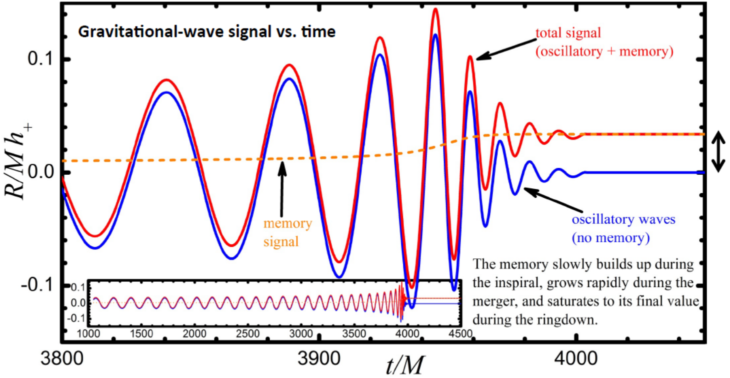


Linear memory [Zeldovich, Polnarev '74][Braginsky, Thorne '87]$\cdots$

- Arises from the non-oscillatory motion of a source, especially due to unbound masses
- E.g. mass/neutrino ejection in supernovas/gamma-ray bursts

Nonlinear memory [Christodoulou '91]$\cdots$

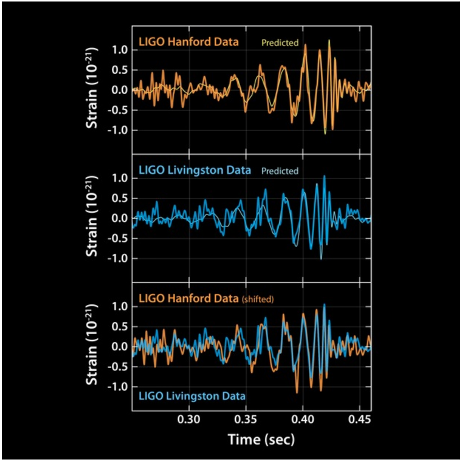
- Arises from the GWs produced by GWs
- Produced by all sources of GWs
- Allows us to probe one of the most nonlinear features of GR



Nonlinear memory from binary black-hole mergers

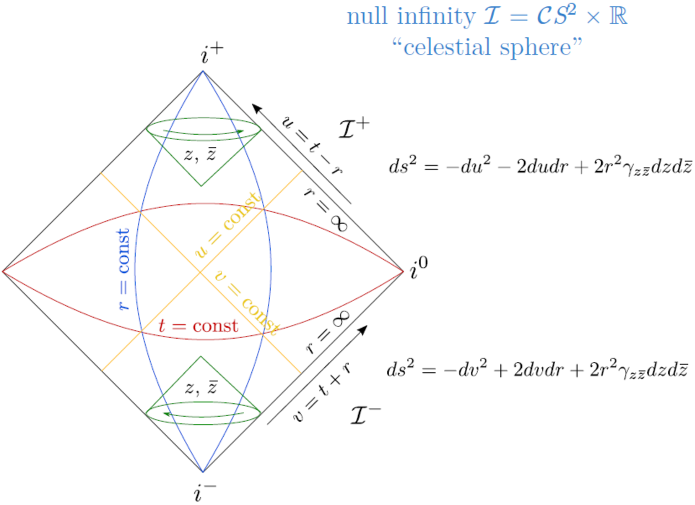
The wave no longer returns to the zero-point of its oscillation.
This growing-offset is called the *memory*.

Detection



- The memory effect is harder to see than gravity waves themselves but has a decent chance of being measured in the coming decades
- A variety of methods of detection of the memory effect has been proposed at LIGO [Lasky *et al.*], via a pulsar timing array [van Haasteren, Levin], etc.

Linear Memory



Metric of flat Minkowski space in retarded coordinates ($u = t - r$) near $\mathcal{I}^+$:

$$ds^2 = -du^2 - 2dudr + 2r^2\gamma_{z\bar{z}}dzd\bar{z}$$

Metric that is asymptotic to, but not exactly equal to, the flat metric?

- Work in Bondi coordinates/gauge $(u, r, z, \bar{z})$, $\Theta^A = (z, \bar{z})$:

$$ds^2 = -Udu^2 - 2e^{2\beta}dudr + g_{AB}\left(d\Theta^A + \frac{1}{2}U^A du\right)\left(d\Theta^B + \frac{1}{2}U^B du\right)$$

  $g_{rr} = g_{rA} = 0$: local diffeomorphism invariance

- Impose asymptotic flatness at large r with fixed $(u, z, \bar{z})$ - boundary falloff conditions on the metric components:

$$\begin{aligned} U &= 1 - \frac{2m_B}{r} + \mathcal{O}(\frac{1}{r^2}), \quad \beta = \mathcal{O}(\frac{1}{r^2}), \\ U_A &= \frac{1}{r^2}D^B C_{BA} + \mathcal{O}(\frac{1}{r^3}), \quad g_{AB} = r^2\gamma_{AB} + rC_{AB} + \mathcal{O}(1) \end{aligned}$$

  where γ_{AB} is the two-dimensional metric on the celestial sphere, $\mathcal{CS}^2$, D_A is the covariant derivative with respect to γ_{AB}.

Natural choice made by Bondi, van der Burg, Metzner, and Sachs (BMS):

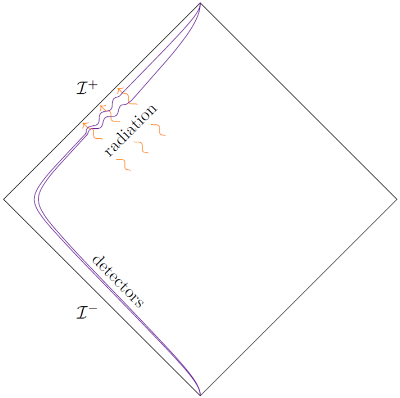
$$
\begin{aligned}
ds^2 = &-du^2 - 2dudr + 2r^2\gamma_{z\bar{z}}dzd\bar{z} \\
&+ \frac{2m_B}{r}du^2 + rC_{zz}dz^2 + rC_{\bar{z}\bar{z}}d\bar{z}^2 + D^zC_{zz}dudz + D^{\bar{z}}C_{\bar{z}\bar{z}}dud\bar{z} \\
&+ \frac{1}{r}\left(\frac{4}{3}\left(N_z + u\partial_z m_B\right) - \frac{1}{4}\partial_z(C_{zz}C^{zz})\right)dudz + c.c. + \cdots
\end{aligned}
$$

Near $\mathcal{I}^+$, spacetime is flat to leading order, retarded time $u = t - r + \cdots$

m_B, N_z, C_{zz} depend on $(u, z, \bar{z})$ but not on r

1. m_B: Bondi mass aspect ($m_B = GM$ for Kerr spacetimes)
2. N_z: angular momentum aspect
3. C_{zz}: perturbation of metric, transverse to direction of propagation of GWs

"Bondi news tensor" $N_{zz} = \partial_u C_{zz}$



- Inertial detectors, e.g. eLISA detectors moving on geodesic orbits
- Memory effect characterises a pair of inertial detectors stationed near $\mathcal{I}^+$ in a region with no Bondi news ($N_{zz} = \partial_u C_{zz} = 0$) at both late and early times
- At intermediate times, gravity waves may pass through, causing oscillating distortions in their relative separations, denoted $(s^z, s^{\bar{z}})$

Geodesic deviation equation for a small perturbation around the flat space:

$$r^2\gamma_{z\bar{z}}\partial_u^2 s^{\bar{z}} = -R_{uzuz}s^z \qquad \left(R_{uzuz} = -\frac{r}{2}\partial_u^2 C_{zz}\right)$$

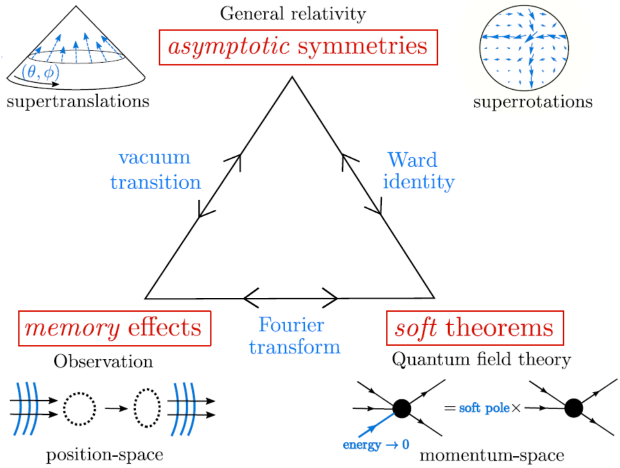
Integrating this equation reveals a DC effect: initial and final separations differ by (in retarded coordinates)

$$\Delta s^{\bar{z}} = \frac{\gamma^{z\bar{z}}}{2r}\Delta C_{zz}s^z$$

gravitational memory effect

The difference ΔC_{zz} between initial and final transverse metric components need not vanish, as flatness does not require $C_{zz} = 0$.

The Infrared Triangle



Deep IR physics is extremely rich!

(I) Memory Effect $\xleftrightarrow{\text{Fourier Transform}}$ Soft Theorem

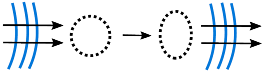
Braginsky-Thorne formula for the gravitational memory effect
(Scattering of black holes)
[V. Braginsky, K. S. Thorne '87]

permanent change in the gravitational-wave field (the burst's memory) $\delta h_{ij}^{\mathrm{TT}}$ is equal to the 'transverse, traceless (TT) part'[36] of the time-independent, Coulomb-type, $1/r$ field of the final system minus that of the initial system. If $\mathbf{P}^A$ is the 4-momentum of mass A of the system and P_i^A is a spatial component of that 4-momentum in the rest frame of the distant observer, and if $\mathbf{k}$ is the past-directed null 4-vector from observer to source, then $\delta h_{ij}^{\mathrm{TT}}$ has the following form:

$$\delta h_{ij}^{\mathrm{TT}} = \delta\left(\sum_A \frac{4P_i^A P_j^A}{\mathbf{k}\cdot\mathbf{P}^A}\right)^{\mathrm{TT}} \qquad (1)$$

Here we use units with $G = c = 1$. In the observer's local Car-

Observation

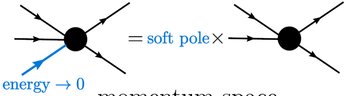


position-space

Weinberg soft graviton theorem
(Scattering of elementary particles)
[S. Weinberg '65]

The dominance of the $1/(p\cdot q)$ pole in (2.5) implies that the effect of attaching one soft-graviton line to an arbitrary diagram is to supply a factor equal to the sum of (2.5) over all external lines in the diagram

$$(8\pi G)^{1/2}\sum_n \eta_n p_n{}^\mu p_n{}^\nu/[p_n\cdot q - i\eta_n\epsilon]. \qquad (2.7)$$



momentum-space

$$\textbf{Memory Effect} \xleftrightarrow{\textsf{Fourier Transform}} \textbf{Soft Theorem}$$

$$\delta h_{ij}^{TT} = \delta \left(\sum_A \frac{4P_i^A P_j^A}{\mathbf{k} \cdot \mathbf{P}^A} \right)^{TT} \qquad (8\pi G)^{1/2} \sum_n \frac{\varepsilon_{\mu\nu} p_n^\mu p_n^\nu}{\mathbf{q} \cdot \mathbf{p}_n}$$

- Replace the four-momenta P_i^A of colliding stars or black holes with the four-momenta p_n^μ of soft graviton
- Account for the different conventions for Newton's constant G and normalisation
- Substitute the graviton momentum $\mathbf{q}$ with its energy ω times the unit null vector $\mathbf{k}$ via $\mathbf{q} = \omega \mathbf{k}$
- Act with a Fourier transform $\int dt\ e^{i\omega t}$ on the Weinberg momentum-space formula to obtain the Braginsky-Thorne formula

$$\textbf{Memory Effect} \xleftrightarrow{\textbf{Fourier Transform}} \textbf{Soft Theorem}$$

- Soft gravitons may seem a bit *unphysical*, because it takes longer and longer to measure them as $E \to 0$.
- Surprise! Memory effect can be measured in a finite time, because the Fourier transform of the Weinberg pole is a step function in retarded time.
- At very long distances, astrophysical black holes and elementary particles are both effectively pointlike!

Universality of IR phenomena

(II) Soft Theorem $\xleftrightarrow{\text{Ward Identity}}$ Asymptotic Symmetry

Conserved charge Q generate supertranslations. In the quantum theory, Q commute with the $\mathcal{S}$-matrix:

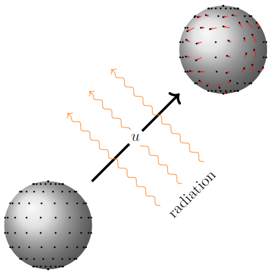
$$Q\mathcal{S} - \mathcal{S}Q = 0.$$

Ward identity

$$\langle \text{out}|Q\mathcal{S}|\text{in}\rangle \sim \text{Weinberg pole} \times \langle \text{out}|\mathcal{S}|\text{in}\rangle$$

is precisely Weinberg's soft graviton theorem.

(III) Asymptotic Symmetry $\xleftrightarrow{\text{Vacuum Transition}}$ Memory Effect

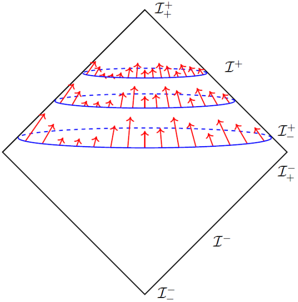


- An array of evenly spaced inertial detectors (black dots) on the sphere $\mathcal{CS}^2$ near $\mathcal{I}^+$ will be permanently displaced (red arrows) by the passage of gravitational radiation
- A pulse of radiation passing through $\mathcal{I}^+$: a *domain wall* connecting two diffeomorphic but BMS-inequivalent vacua that are related by an asymptotic symmetry
- Displacements/supertranslations: measurement of the BMS diffeomorphism, which relates the vacua before and after the passage of the radiation

[Bondi, van der Burg, Metzner, and Sachs '62]

Supertranslations [Bondi, van der Burg, Metzner, and Sachs '62]

$$\mathcal{L}_f C_{zz} = f\partial_u C_{zz} - 2D_z^2 f$$



No energy flux or retarded time dependence of the asymptotic data at early and late times:

$$\partial_u C_{zz}^{\text{early}} = 0 = \partial_u C_{zz}^{\text{late}}$$

BMS vacuum transition: early and late geometries are related by a supertranslation

$$\Delta C_{zz} = C_{zz}^{\text{late}} - C_{zz}^{\text{early}} = -2D_z^2 f,$$
$$f = \mathcal{L}_f C(z,\bar{z})$$

$C(z,\bar{z})$ is the Goldstone boson of spontaneously broken supertranslation invariance.

$$\Delta C_{zz} = C_{zz}^{\text{late}} - C_{zz}^{\text{early}} = -2D_z^2 f,$$

where

$$f = \int d^2w \ \gamma_{w\bar{w}} G(z,\bar{z};w,\bar{w}) \left(\int_{u_i}^{u_f} du \ T_{uu}(w,\bar{w}) + \Delta m_B \right)$$

uu stress tensor:

$$T_{uu} = \frac{1}{4} N_{zz} N^{zz} + 4\pi G \lim_{r\to\infty} \left(r^2 T_{uu}^R \right)$$

Green's function:

$$G(z,\bar{z};w,\bar{w}) = \frac{1}{\pi} \sin^2 \frac{\Delta\Theta}{2} \log \left(\sin^2 \frac{\Delta\Theta}{2} \right) \quad \left(\sin^2 \frac{\Delta\Theta}{2} = \frac{|z-w|^2}{(1+z\bar{z})(1+w\bar{w})} \right)$$

with $\Delta\Theta$ the angle on $\mathcal{CS}^2$ between $(z,\bar{z})$ and $(w,\bar{w})$
Change in proper distance between the detectors:

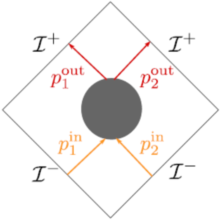
$$\Delta L = \frac{r_0}{2L_0} \left[\Delta C_{zz}(z_0,\bar{z}_0) \delta z^2 + \Delta C_{\bar{z}\bar{z}}(z_0,\bar{z}_0) \delta \bar{z}^2 \right]$$

supertranslation induced by GWs passing through $\mathcal{I}^+$

Memory Effect $\xleftrightarrow{\textbf{Vacuum Transition}}$ Asymptotic Symmetry

Remark: *Celestial / flat space holography*

Bulk Mink[1,3]	**Boundary $\mathcal{CS}^2$**
4D $SL(2,\mathbb{C})$ Lorentz	2D global conformal
4D superrotations	2D local conformal
4D supertranslations	2D Kac-Moody



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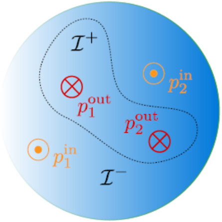

4D $\mathcal{S}$-matrix	2D conformal correlator
$\langle out\vert\mathcal{S}\vert in\rangle$	$\langle\mathcal{O}_1(z_1,\bar{z}_1)\dots\mathcal{O}_n(z_n,\bar{z}_n)\rangle$
$\omega,\vec{p},\ell$	$(z,\bar{z}),\ \Delta = h+\bar{h},\ J = h-\bar{h}$

Thanks!